

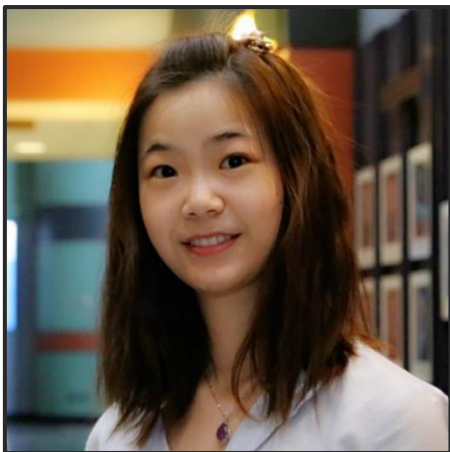


# Angular Visual Hardness

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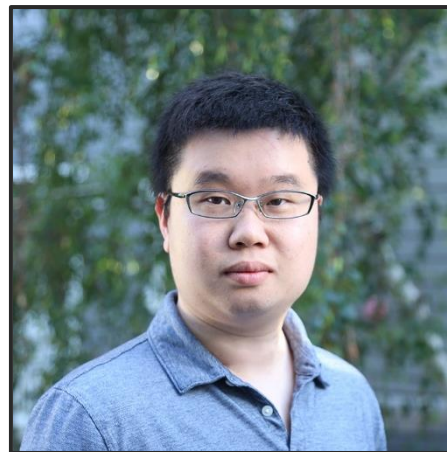




Beidi Chen, Rice



Weiyang Liu, Georgia Tech



Zhiding Yu, NVIDIA



Jan Kautz, NVIDIA



Anshumali Shrivastava, Rice



Animesh Garg, NVIDIA



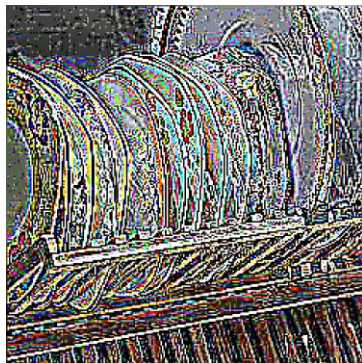
Anima Anandkumar, NVIDIA

# Human Visual Hardness

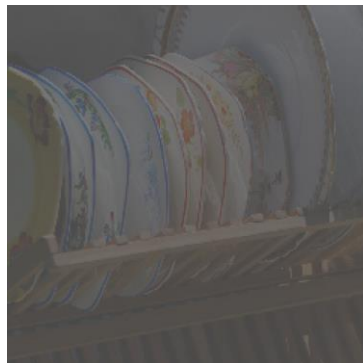
plate rack



sharpness



contrast



blur

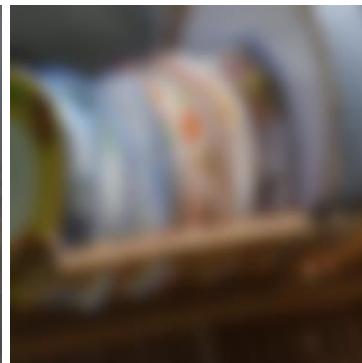


Image Degradation

dishwasher



saltshaker



nail



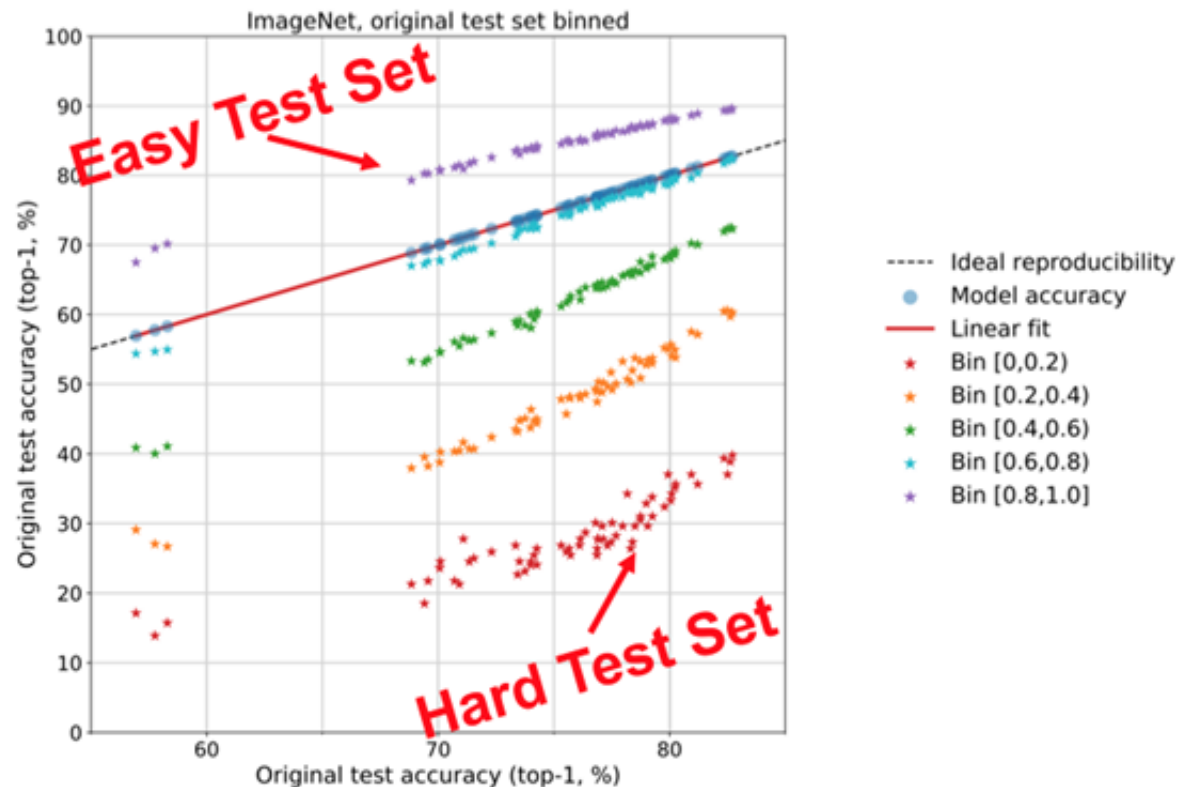
oil filter



Semantic Ambiguity

# Human Selection Freq (HSF): A Visual Hardness Proxy

## Human Labeling Interface





# Gap between Human Recognition and CNNs

**Hard** for Human but **Easy** for CNNs



Nail

**Softmax** 0.93

**HSF** 0.2



Oil Filter

0.998

0.2

**Easy** for Human but **Hard** for CNNs



Golf Ball

0.001

1.0



Radio

0.001

1.0

# Softmax Cross-Entropy Loss

$$L = \frac{1}{N} \sum_i L_i = \frac{1}{N} \sum_i -\log \left( \frac{e^{f_{y_i}}}{\sum_j e^{f_j}} \right)$$

Magnitude Information

Angle Information

$$L_i = -\log \left( \frac{e^{\|W_{y_i}\| \|\mathbf{x}_i\| \cos(\theta_{y_i})}}{\sum_j e^{\|W_j\| \|\mathbf{x}_i\| \cos(\theta_j)}} \right)$$

Model Confidence

The diagram illustrates the decomposition of the Softmax Cross-Entropy Loss into magnitude and angle information. The loss function is shown as  $L_i = -\log \left( \frac{e^{\|W_{y_i}\| \|\mathbf{x}_i\| \cos(\theta_{y_i})}}{\sum_j e^{\|W_j\| \|\mathbf{x}_i\| \cos(\theta_j)}} \right)$ . A blue box highlights the term  $\|W_{y_i}\| \|\mathbf{x}_i\|$ , which is labeled 'Magnitude Information'. A red box highlights the term  $\cos(\theta_{y_i})$ , which is labeled 'Angle Information'. A magenta box encloses the entire fraction, which is labeled 'Model Confidence'.

# Angular Visual Hardness (AVH)

Given a sample  $x$  with label  $y$ :

$$AVH(x) = \frac{\mathcal{A}(x, w_y)}{\sum_{i=1}^C \mathcal{A}(x, w_i)}$$

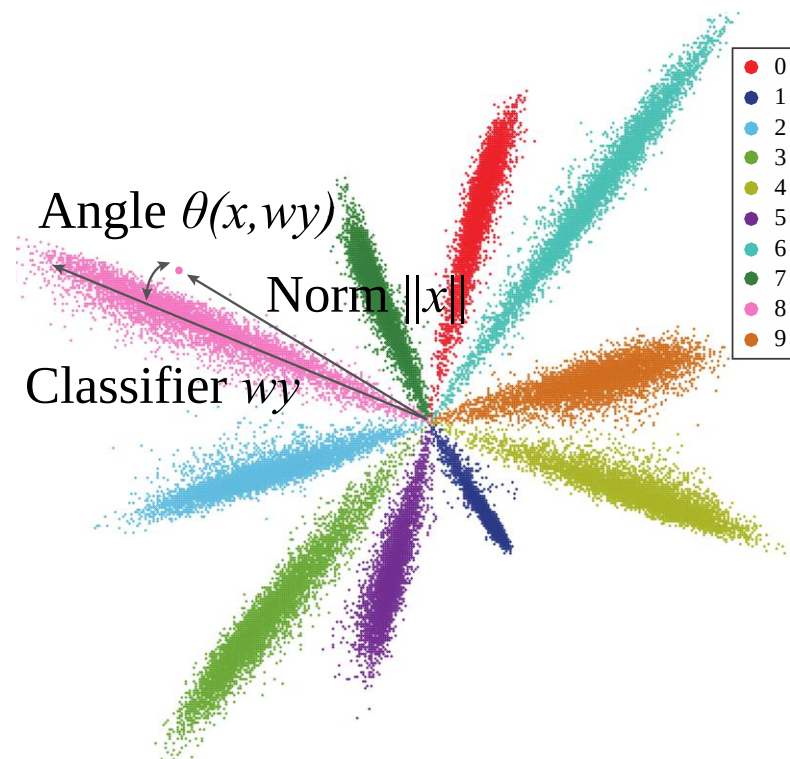
where,

$$\mathcal{A}(u, v) = \arccos\left(\frac{\langle u, v \rangle}{\|u\| \|v\|}\right)$$

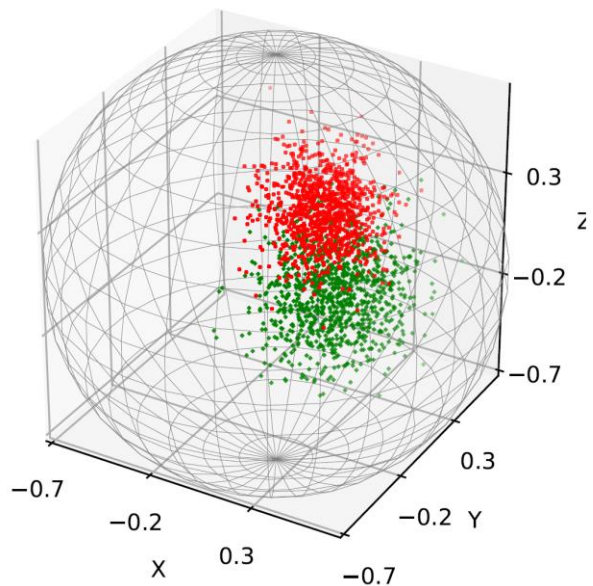
$w_i$  is the classifier for the  $i$ -th class.

**Theoretical Foundation:**

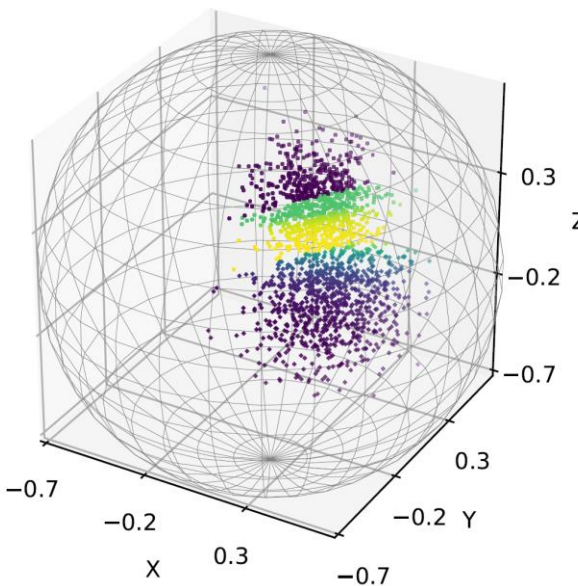
Soudry et al, The Implicit Bias of Gradient Descent on Separable Data, ICLR18



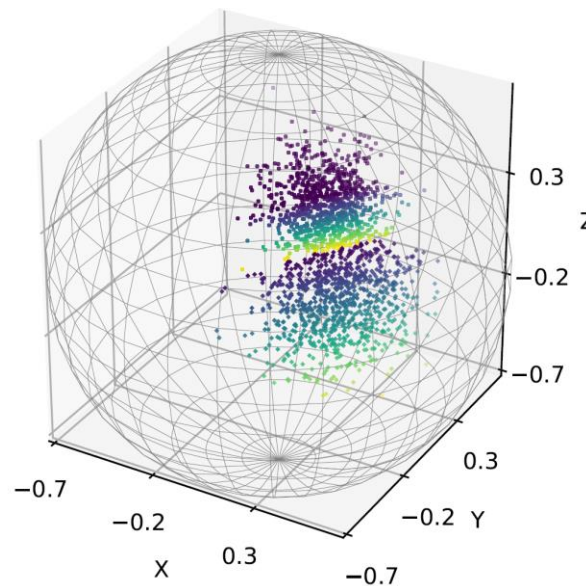
# Toy Example: AVH vs. $||x||$



**Raw data**



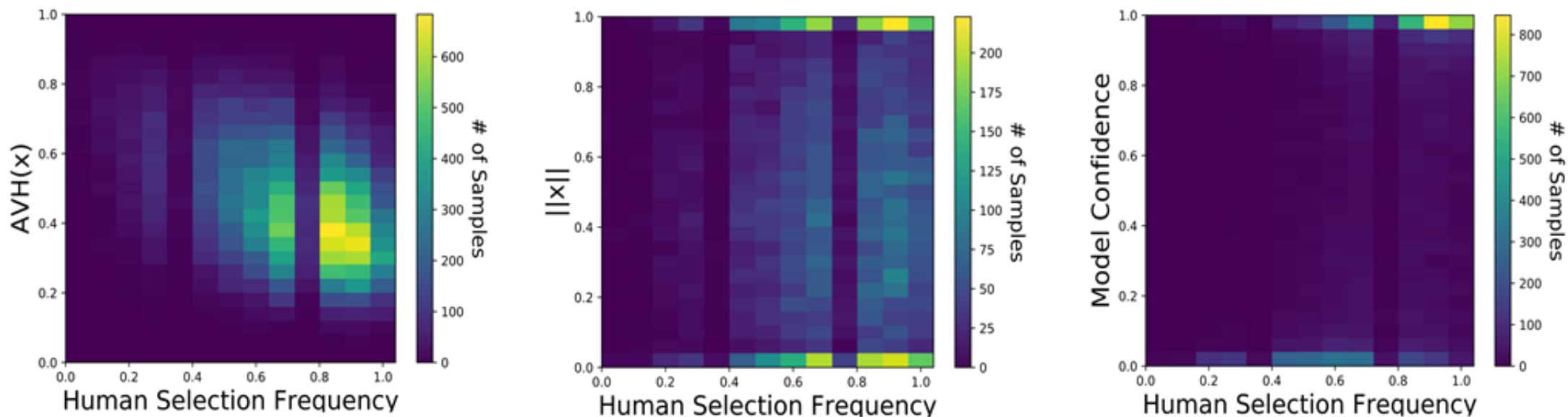
**Heat map of AVH**



**Heat map of  $||x||$**



# Correlation between Different Measures and HSF

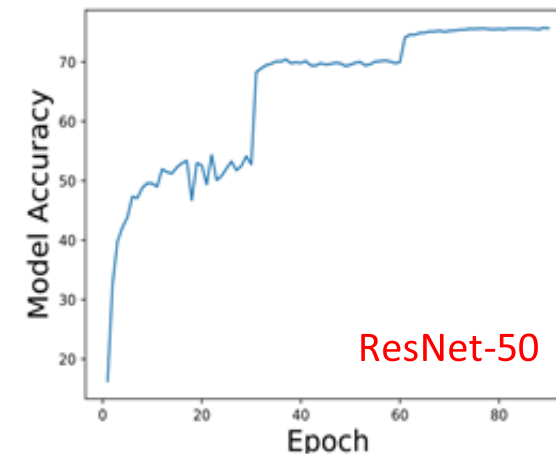
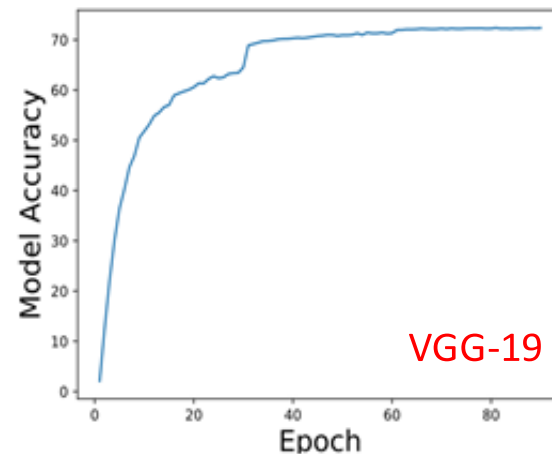
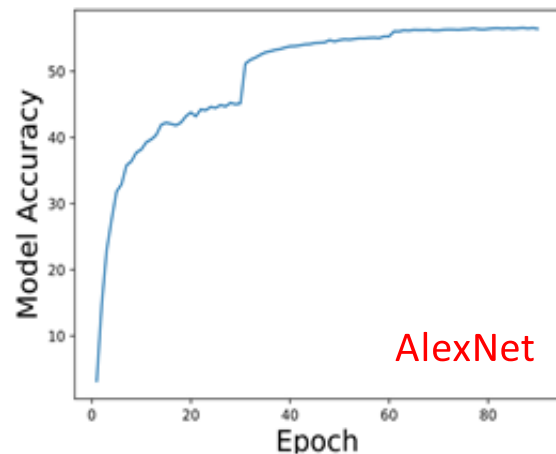
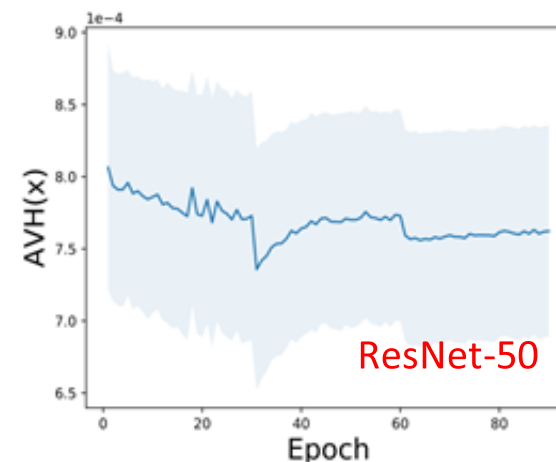
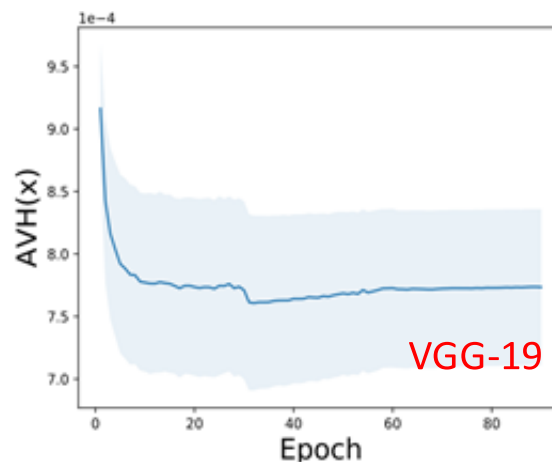
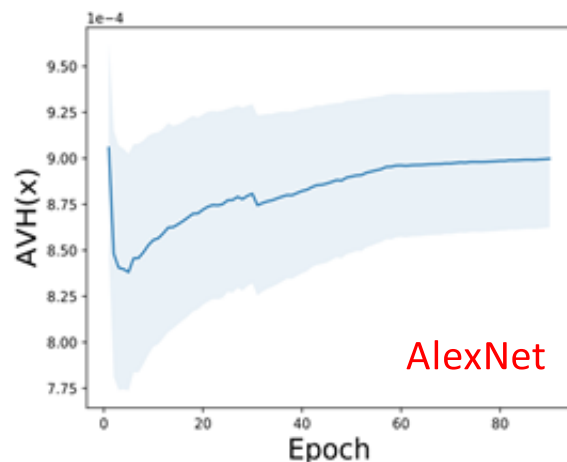


Spearman rank correlations

|                   | z-score | Total Coef | [0, 0.2] | [0.2, 0.4] | [0.4, 0.6] | [0.6, 0.8] | [0.8, 1.0] |
|-------------------|---------|------------|----------|------------|------------|------------|------------|
| Number of Samples | -       | 29987      | 837      | 2732       | 6541       | 11066      | 8811       |
| AVH               | 0.377   | 0.36       | 0.228    | 0.125      | 0.124      | 0.103      | 0.094      |
| Model Confidence  | 0.337   | 0.325      | 0.192    | 0.122      | 0.102      | 0.078      | 0.056      |
| $\ x\ _2$         | -       | 0.0017     | 0.0013   | 0.0007     | 0.0005     | 0.0004     | 0.0003     |

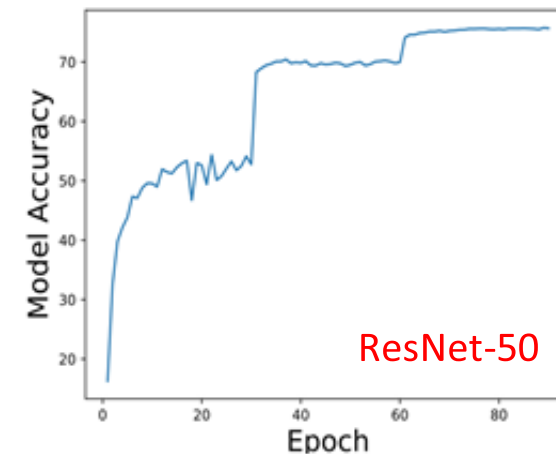
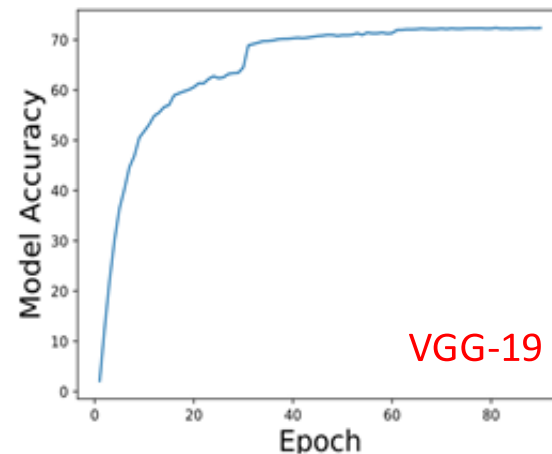
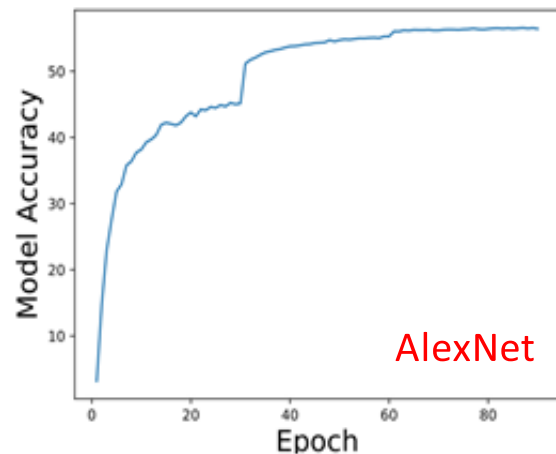
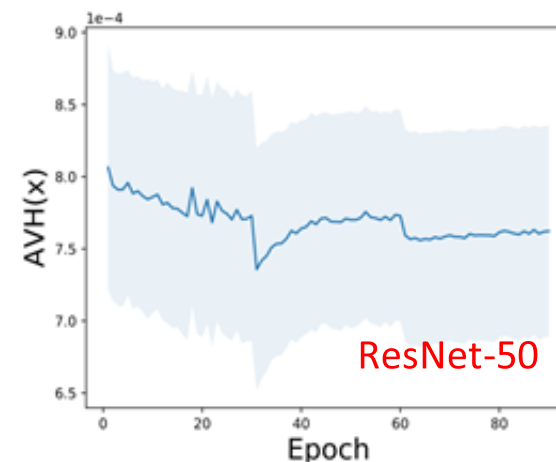
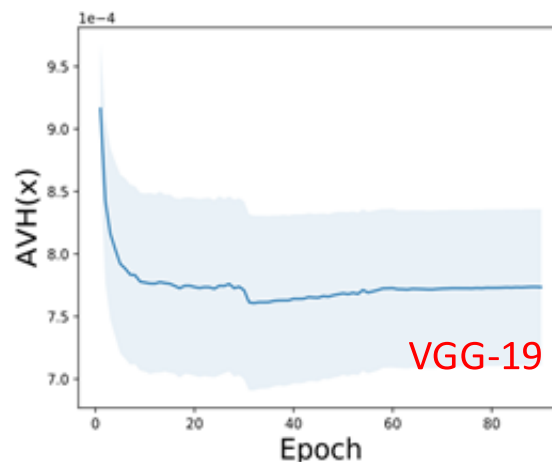
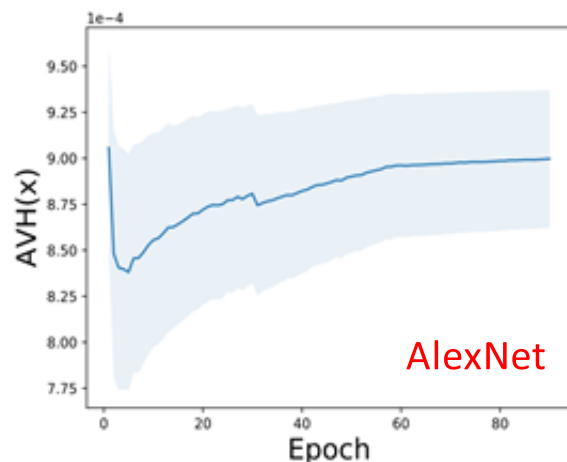
# Main Discoveries

**Discovery 1** - AVH hits plateau early even though accuracy or loss is still improving



# Main Discoveries

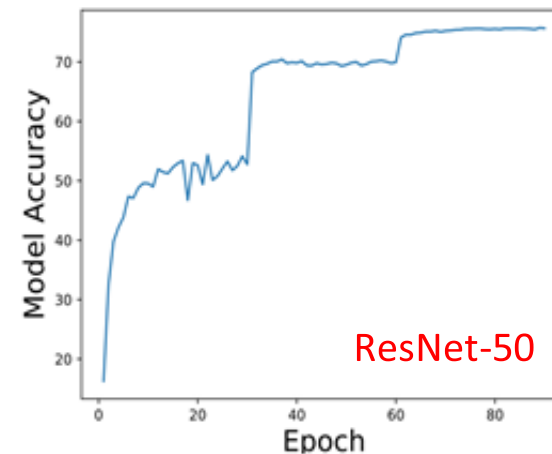
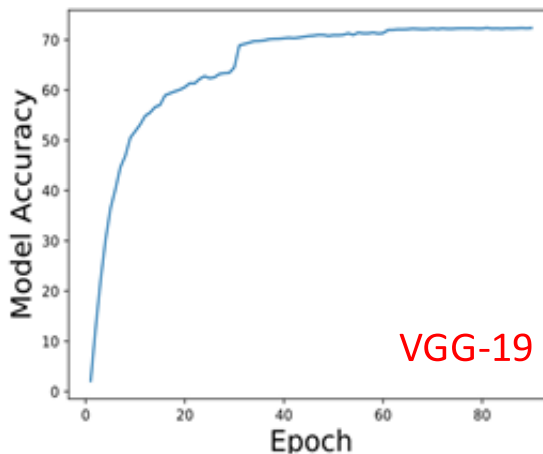
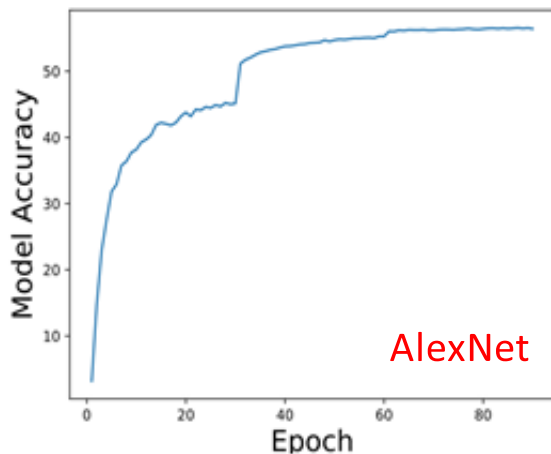
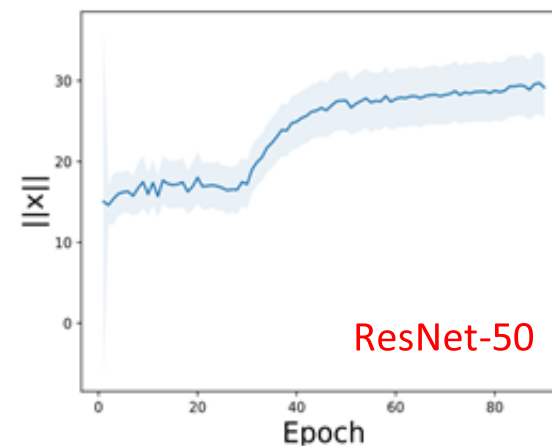
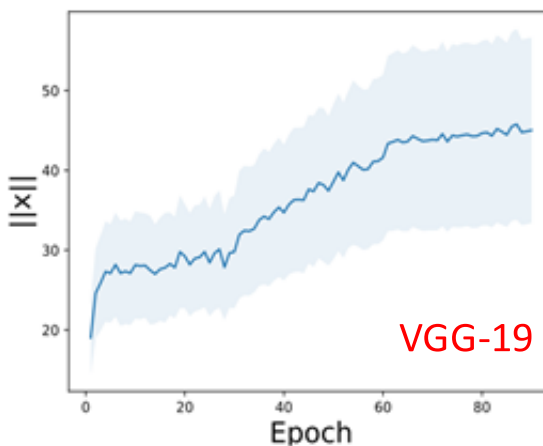
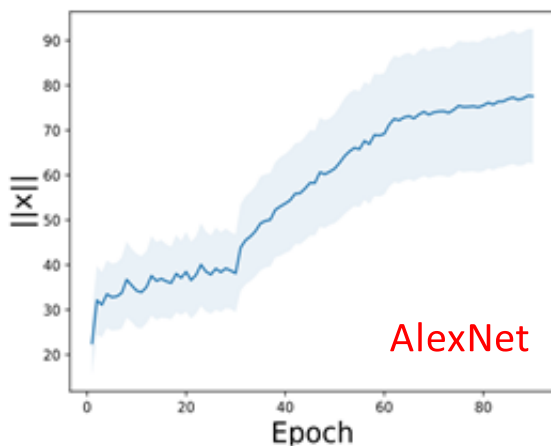
## Discovery 2 - AVH is an indicator of model's generalization ability





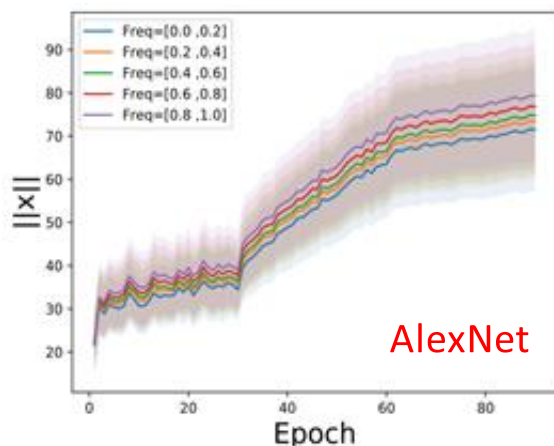
# Main Discoveries

**Discovery 3** - The norm of feature embeddings keeps increasing during training

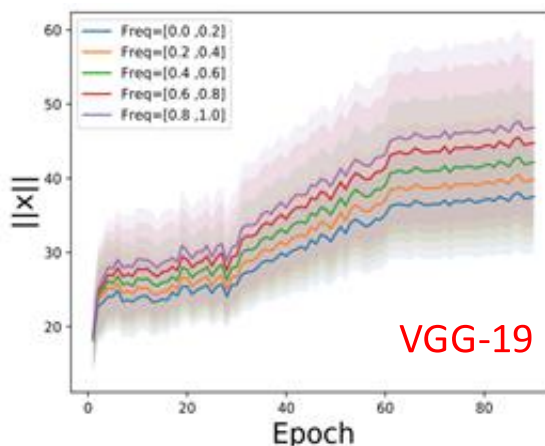


# Main Discoveries

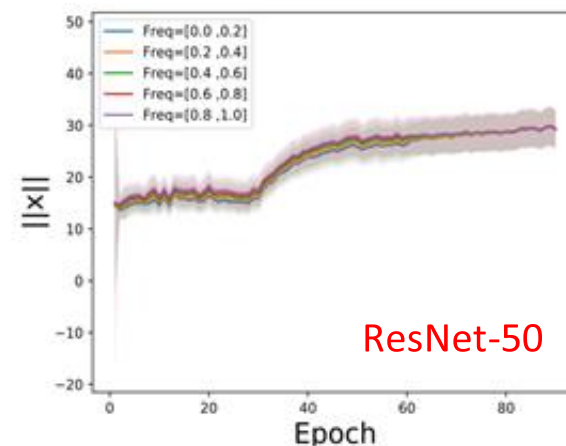
## Discovery 4 - Correlation between AVH and human selection freq holds across models



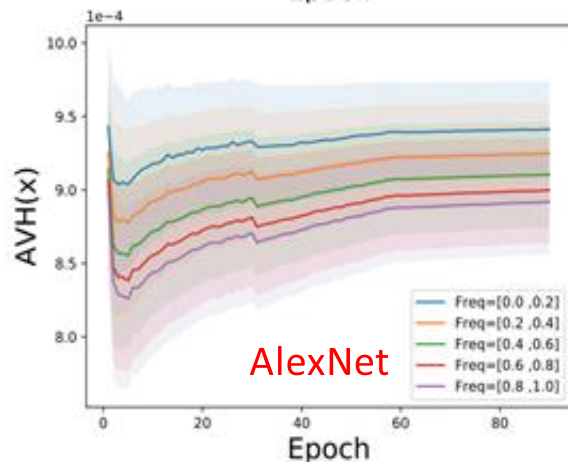
AlexNet



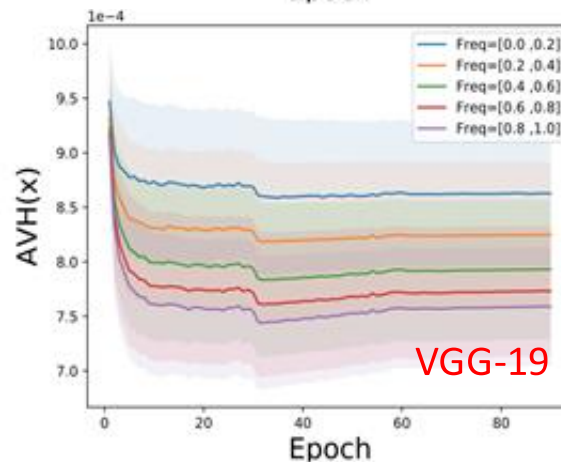
VGG-19



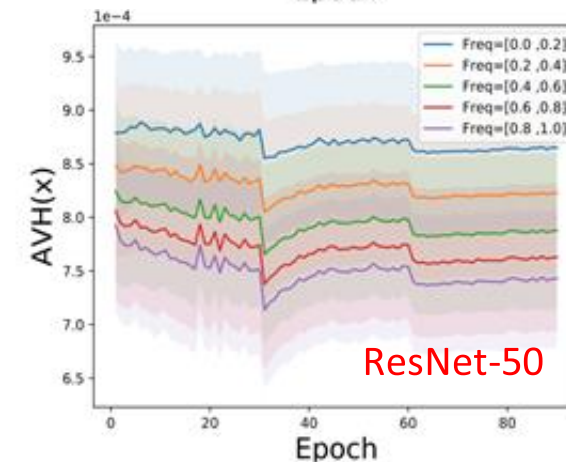
ResNet-50



AlexNet



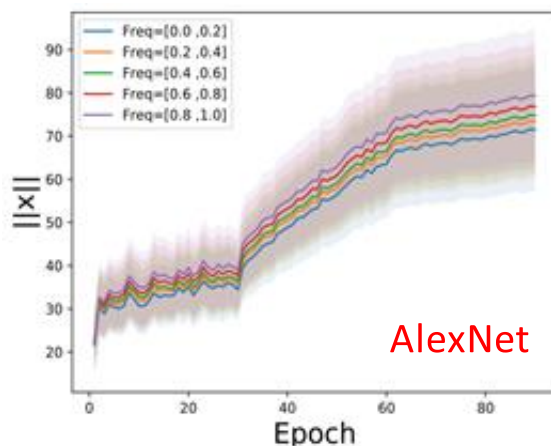
VGG-19



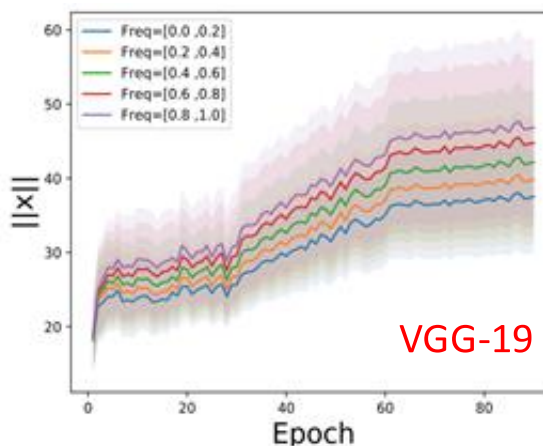
ResNet-50

# Main Discoveries

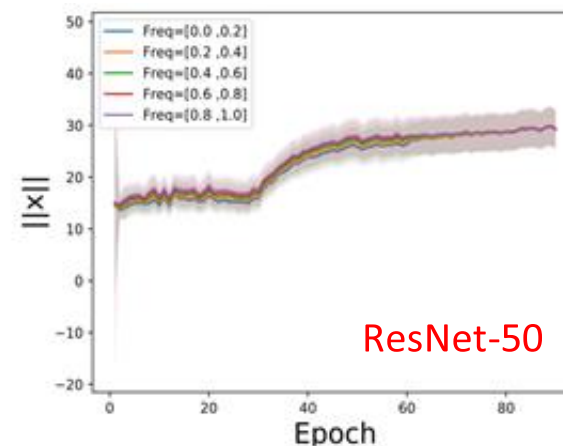
**Discovery 5** - Correlation between norm and human selection frequency is not consistent



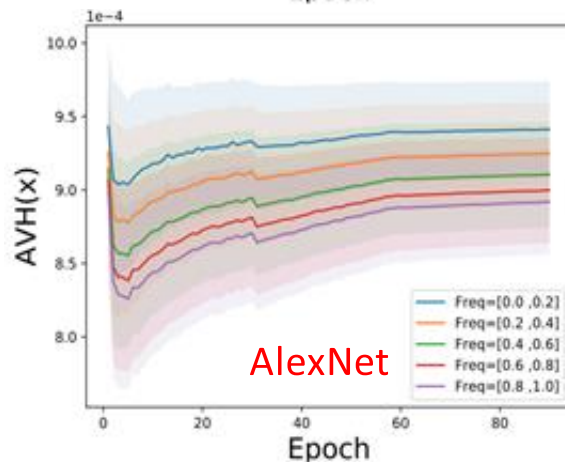
AlexNet



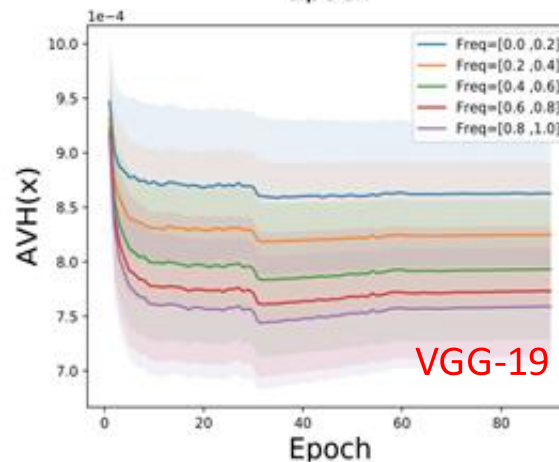
VGG-19



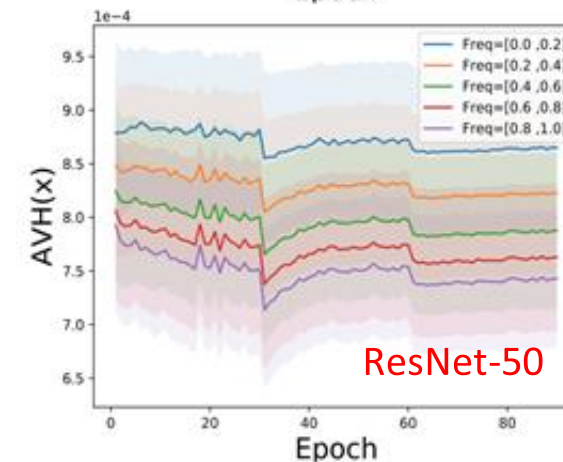
ResNet-50



AlexNet



VGG-19



ResNet-50

# Conjecture on training dynamic of CNNs

- Softmax cross-entropy loss will first optimize the angles among different classes while the norm will fluctuate and increase very slowly.
- The angles become more stable and change very slowly while the norm increases rapidly.
- Easy examples: the angles get decreased enough for correct classification, the softmax cross-entropy loss can be well minimized by increasing the norm.
- Hard examples: the plateau is caused by being unable to decrease the angle to correctly classify examples or increase the norms otherwise hurting loss.



# Application I: Self-Training for Domain Adaptation

VisDA17  
Dataset



Car

Source Domain (Labeled)

Adaptation



Target Domain (Unlabeled)

CBST

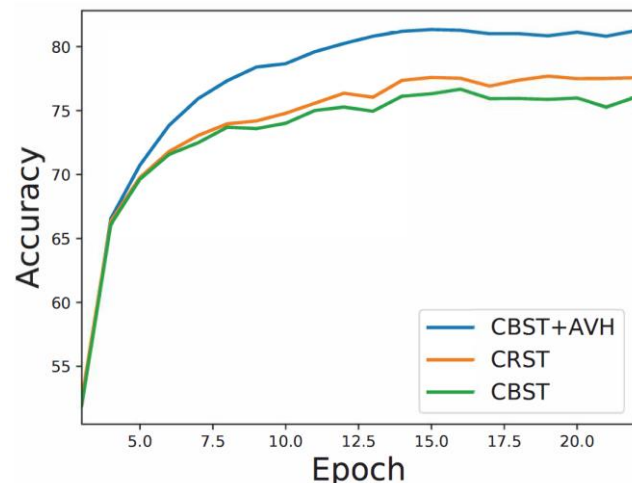
$$\hat{y}_t^{(k)*} = \begin{cases} 1, & \text{if } k = \arg \max_c \left\{ \frac{p(c|\mathbf{x}_t; \mathbf{w})}{\lambda_c} \right\} \\ & \text{and } p(k|\mathbf{x}_t; \mathbf{w}) > \lambda_k \\ 0, & \text{otherwise} \end{cases}$$

CBST + AVH

$$\mathcal{AVC}(c|\mathbf{x}; \mathbf{w}) = \frac{\pi - \mathcal{A}(\mathbf{x}, \mathbf{w}_c)}{\sum_{k=1}^K (\pi - \mathcal{A}(\mathbf{x}, \mathbf{w}_k))}$$
$$\hat{y}_t^{(k)*} = \begin{cases} 1, & \text{if } k = \arg \max_c \left\{ \frac{p(c|\mathbf{x}_t; \mathbf{w})}{\lambda_c} \right\} \\ & \text{and } \mathcal{AVC}(k|\mathbf{x}_t; \mathbf{w}) > \beta_k \\ 0, & \text{otherwise} \end{cases}$$

Improved  
selection

# Application I: Self-Training for Domain Adaptation

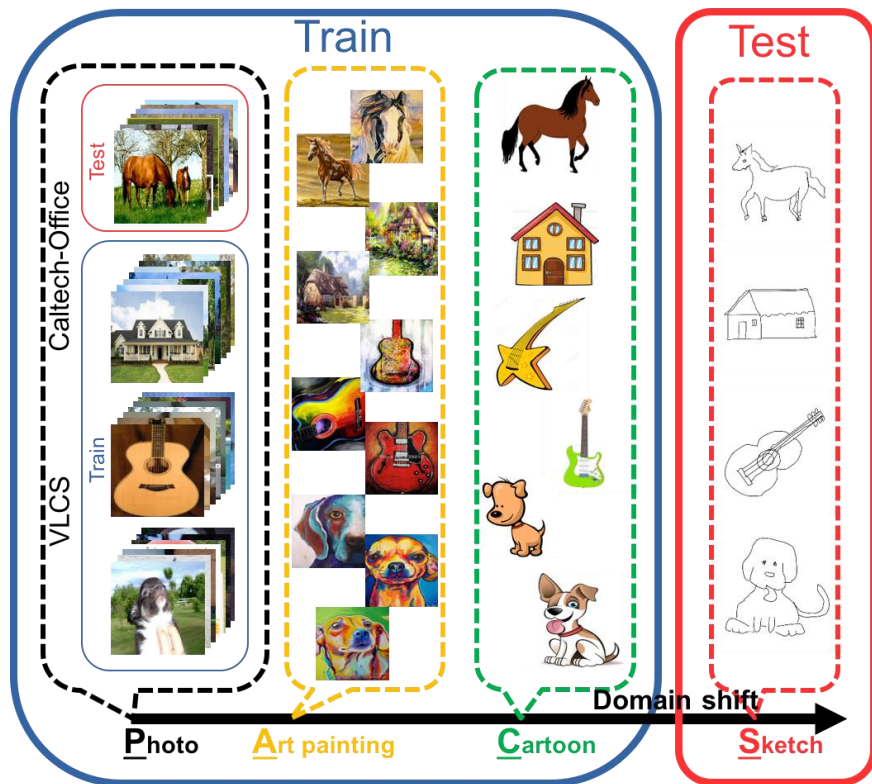


Examples chosen by  
AVH but not Softmax

| Method                          | Aero        | Bike        | Bus         | Car         | Horse       | Knife       | Motor       | Person      | Plant       | Skateboard  | Train       | Truck       | Mean        |
|---------------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Source (Saito et al., 2018)     | 55.1        | 53.3        | 61.9        | 59.1        | 80.6        | 17.9        | 79.7        | 31.2        | 81.0        | 26.5        | 73.5        | 8.5         | 52.4        |
| MMD (Long et al., 2015b)        | 87.1        | 63.0        | 76.5        | 42.0        | 90.3        | 42.9        | 85.9        | 53.1        | 49.7        | 36.3        | <b>85.8</b> | 20.7        | 61.1        |
| DANN (Ganin et al., 2016)       | 81.9        | 77.7        | 82.8        | 44.3        | 81.2        | 29.5        | 65.1        | 28.6        | 51.9        | 54.6        | 82.8        | 7.8         | 57.4        |
| ENT (Grandvalet & Bengio, 2005) | 80.3        | 75.5        | 75.8        | 48.3        | 77.9        | 27.3        | 69.7        | 40.2        | 46.5        | 46.6        | 79.3        | 16.0        | 57.0        |
| MCD (Saito et al., 2017b)       | 87.0        | 60.9        | <b>83.7</b> | 64.0        | 88.9        | 79.6        | 84.7        | 76.9        | 88.6        | 40.3        | 83.0        | 25.8        | 71.9        |
| ADR (Saito et al., 2018)        | 87.8        | 79.5        | <b>83.7</b> | 65.3        | <b>92.3</b> | 61.8        | <b>88.9</b> | 73.2        | 87.8        | 60.0        | 85.5        | 32.3        | 74.8        |
| Source (Zou et al., 2019)       | 68.7        | 36.7        | 61.3        | <b>70.4</b> | 67.9        | 5.9         | 82.6        | 25.5        | 75.6        | 29.4        | 83.8        | 10.9        | 51.6        |
| CBST (Zou et al., 2019)         | 87.2        | 78.8        | 56.5        | 55.4        | 85.1        | 79.2        | 83.8        | 77.7        | 82.8        | <b>88.8</b> | 69.0        | <b>72.0</b> | 76.4        |
| CRST (Zou et al., 2019)         | 88.0        | 79.2        | 61.0        | 60.0        | 87.5        | 81.4        | 86.3        | 78.8        | 85.6        | 86.6        | 73.9        | 68.8        | 78.1        |
| Proposed                        | <b>93.3</b> | <b>80.2</b> | 78.9        | 60.9        | 88.4        | <b>89.7</b> | <b>88.9</b> | <b>79.6</b> | <b>89.5</b> | 86.8        | 81.5        | 60.0        | <b>81.5</b> |

# Application II: AVH Loss for Domain Generalization

PACS Dataset



$$\mathcal{L}_{AVH} = \sum_i \frac{\exp(s(\pi - \mathcal{A}(\mathbf{x}_i, \mathbf{w}_{y_i})))}{\sum_{k=1}^K \exp(s(\pi - \mathcal{A}(\mathbf{x}_i, \mathbf{w}_k)))}$$

| Method                           | Painting     | Cartoon      | Photo        | Sketch       | Avg          |
|----------------------------------|--------------|--------------|--------------|--------------|--------------|
| AlexNet (Li et al., 2017)        | 62.86        | 66.97        | 89.50        | 57.51        | 69.21        |
| MLDG (Li et al., 2018)           | 66.23        | 66.88        | 88.00        | 58.96        | 70.01        |
| MetaReg (Balaji et al., 2018)    | <b>69.82</b> | 70.35        | <b>91.07</b> | 59.26        | <b>72.62</b> |
| Feature-critic (Li et al., 2019) | 64.89        | <b>71.72</b> | 89.94        | <b>61.85</b> | 72.10        |
| Baseline CNN-9                   | 66.46        | 67.88        | 89.70        | 51.72        | 68.94        |
| CNN-9 + AVH                      | <b>71.56</b> | <b>69.25</b> | <b>89.93</b> | <b>60.86</b> | <b>72.90</b> |

**Thanks You!**